A Two-Queue Model with Alternating Limited Service and State-Dependent Setups

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Stochastic Economic Lot Scheduling Problem

- Multiple products;
- Single machine;
- Only one product can be produced at a time;
- Bottleneck (expensive machine);
- Holding costs;
- Backorder costs;
- Setup costs and setup times;
- Make-to-stock.
Goal

Minimization of total costs:

1. Holding costs;
2. Backorder costs;
3. Setup costs.

- *short cycles*: lead to frequent production opportunities;
- *large cycles*: decrease setup frequencies.
Base-stock policy

Federgruen and Katalan [1996, 1998]

• *Fixed production sequence*:
  – Order and frequency in which products are produced.

• *Base-stock policy*:
  – Continue production until the target inventory level $b_i$ is reached.

Drawbacks

• Setups are incurred even when there is no shortfall;

• One product may occupy the machine for a long period of time.
Base-stock policy

Federgruen and Katalan [1996, 1998]

• Fixed production sequence:
  – Order and frequency in which products are produced.

• Base-stock policy:
  – Continue production until the target inventory level $b_i$ is reached.

Drawbacks

• Setups are incurred even when there is no shortfall (state-dependent setups);

• One product may occupy the machine for a long period of time (quantity-limited base-stock policy, i.e., produce until either the target inventory level $b_i$ is reached or a maximum number of items has been produced).
Overview

- Model;
- Analysis;
- Numerical evaluation;
- Future research.
Model

- 2 independent products;
- Demand for item $i$ follows a Poisson process with rate $\lambda_i$;
- Individual production times are generally distributed;
- Setup times are generally distributed;
- Setups are state-dependent;
- When the system is idle, the machine is turned off;
- Item 1 is produced according to a standard base-stock policy;
- Item 2 is produced according to the quantity-limited base-stock policy.

The goal of our research is to get more insight in the effect of the quantity-limited base-stock policy on the total costs.
Total costs

- \( K_i \): setup costs of item \( i \);
- \( h_i \): holding costs of item \( i \);
- \( p_i \): backorder costs of item \( i \).

\[
F = \sum_{i=1}^{2} \frac{K_i}{E[C]} + \sum_{i=1}^{2} E[h_i I_i^+ + p_i I_i^-],
\]

with \( E[C] \) the mean cycle length and \( I_i(t) \) the inventory level of item \( i \).
Specific realization

\[ I_i(t) = b_i - L_i(t), \quad t \geq 0, \quad i = 1, 2, \]

where \( L_i(t) \) represents the shortfall of station \( i \) at time \( t \).
Optimal base-stock levels

\[ F = \sum_{i=1}^{2} \frac{K_i}{E[C]} + \sum_{i=1}^{2} E[h_i I_i^+ + p_i I_i^-] \]

\[ = \sum_{i=1}^{2} \frac{K_i}{E[C]} + \sum_{i=1}^{2} (h_i E[(b_i - L_i)^+] + p_i E[(L_i - b_i)^+]). \]

- Since the distribution of \( L_i \) is independent of \( b_i \), the optimal target inventory level \( b_i^* \) for given \( k \) is as follows (newsboy problem)

\[ b_i^* = \min \{ n \in \mathbb{N} | P[L_i \leq n] \geq \frac{p_i}{p_i + h_i} \}. \]
Optimization procedure

1. Compute the shortfall, and thus the stock level, distribution for given $k$;
2. Compute optimal base-stock levels given this distribution;
   - Solution of newsboy equations.
Derivation shortfall distributions

Derivation queue sizes distributions for given $k$

Distribution of queue size $L_i$ is derived in the paper by a generating function approach.
Optimization procedure

1. Compute the shortfall, and thus the stock level, distribution for given $k$;
   - Generating function approach.
2. Compute optimal base-stock levels given this distribution;
   - Solution of newsboy equations.
Numerical evaluation (I)

(1. exhaustive 2. k-limited)

\[ \lambda_1 = \lambda_2 = 0.45, \quad \beta_1 = \beta_2 = 1 \quad (\text{mean production time}), \]
\[ p_1 = 2, \quad h_1 = 1, \quad p_2 = 1, \quad h_2 = 0.5. \]
Numerical evaluation (II)

(I. exhaustive  2. k-limited)

\[ \lambda_1 = 0.675, \quad \lambda_2 = 0.225, \quad \beta_1 = \beta_2 = 1, \quad p_1 = p_2 = 1.5, \quad h_1 = h_2 = 0.75. \]
Conclusion

(1. exhaustive  2. k-limited)

• $k$ increases:
  – Costs and base-stock level of item 1 increase;
  – Costs and base-stock level of item 2 decrease.

• $\rho_1$ increases or $\rho_2$ decreases:
  – Base-stock level of item 1 increases;
  – Base-stock level of item 2 decreases.

• $\rho_1$ decreases or $\rho_2$ increases:
  – Base-stock level of item 1 decreases;
  – Base-stock level of item 2 increases.
Future research

- $\geq 2$ products;
- optimization of $k$;
- multi-echelon;
- .....