Performance Analysis of DS Spread Spectrum System in Multiple Narrowband Interference

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Abstract: - The investigation of spectrum overlay of a Spread Spectrum system on the existing narrowband FM broadcasting system is presented. The Spread Spectrum (SS) system is assumed to utilize direct sequence (DS) spreading, using maximal length pseudorandom sequences with long spreading codes. We studied the performance analysis of the SS system due to the overlaying analog FM system, consisting of multiple narrowband FM stations. The SS signal is evaluated for various types of spreading scenarios, for different carrier frequency differences $\Delta f$ and for various signal-to-interference ratios.

Key-Words: - Spread Spectrum, Direct Sequence (DS), Analog FM, AWGN, Interference, Bit-Error-Rate.

1 Introduction

Spread Spectrum has been under development for the last 50 years. The interest in spread spectrum for civilian applications is growing, particularly in the area of mobile communications. One of many reasons for this growing interest is the huge demand of spectrum that has become more and more requisite for modern communication systems. Antijamming, antinterference, privacy and low power spectral density are some of the advantages that the SS technique encompasses and strengthen their use to overlay schemes. Both field tests and analyses have provided a perspective as to what the capabilities of such a system are.

The mutual interference, which in the conditions of high density of these systems becomes unavoidable, through the estimation of the interference noise, serves as a measure for accepting their coexistence. This overlay concept has been demonstrated in both PCS band [1] and cellular band [2]. Reference [3] has proposed allowing a CDMA network to be overlaid on top of existing microwave narrow-band users that occupy a part of the spread-spectrum system bandwidth (BW). In fact, this is another advantage of CDMA over either TDMA or FDMA, because this will increase the overall spectrum capability. However, such an application must be considered carefully, because the CDMA (wide-band) users and the narrow-band users can interfere with one another. This scenario is of particular interest in a CDMA cellular system, in which a given geographical region is divided into many small cells, where the number of cells depends on the size and geometry of the region. Each user has a unique pseudonoise (PN) sequence and communicates through central base-stations with other users in the same cell or different cells. The base station is assumed to control the average transmission power within its particular cell, to overcome the near/far problem.

This spectrum overlay can increase communications capacity and spectral efficiency, but may cause the following types of interference: i) interference from the narrowband FM stations to the SS system and ii) interference from the overlaid wideband SS system on the FM receivers. The first is the scope of this paper.

2 Description Of Spread Spectrum System

The spread spectrum signal is given by [4] as

$$s_r(t) = \sqrt{2P} b(t-\tau) c(t-\tau) \cos(\omega_o t + \theta)$$

where $P$ is the received power of the spread spectrum, $\tau$ is the time delay of the signal uniformly distributed into $[0,T]$, $\theta$ is the phase angle uniformly distributed on $[0,2\pi]$, $b(t)$ is the modulating digital signal given by $b(t) = \sum_{n=-\infty}^{\infty} a_n g_T(t-nT_s)$ where $\{a_n = \pm 1, -\infty < n < +\infty\}$ and $g_T(t)$ is the rectangular pulse of $T_s$ duration, with $P[b(t)=1]=P[b(t)=-1]=0.5$, and $\omega_o$ is the carrier frequency of the signal. $c(t)$ is
the spreading code given by 
\[
c(t) = \sum_{n=-\infty}^{\infty} c_n p(t-nT_c)
\]
where \(c_n\) is one chip of random binary sequence \(\{c_n\}\), which consists of independent symbols with equal probability, \(T_c\) is the spreading code chip duration and \(p(t)\) is the chip waveform, assumed rectangular.

The autocorrelation function of the spreading sequence is given by [5]:

\[
R(\tau) = \begin{cases} 1 - \frac{\tau}{T_c}, & |\tau| < T_c \\ 0, & |\tau| \geq T_c \end{cases}
\]

and its power spectral density by

\[
S_R(f) = \int_{-\infty}^{\infty} R(\tau) \exp(-j2\pi f \tau) d\tau = T_c \text{sinc}^2(\pi f T_c)
\]

where \(f_c=1/T_c\) is the spreading code rate. Perfect code, phase and symbol synchronization are assumed.

### 3 FM Power Spectrum Approximation

The interference power of the FM spectrum at the input of the digital receiver is required in closed-form mathematical expression. The required power spectral densities of the interference FM signals are presented as a result of laboratory measurements and mathematical derivations.

The FM signal is generally a random process and consequently can be only statistically. Specifically, to characterize the power spectral density of the FM signal we must measure the values of statistical parameters, such as mean, maximum, minimum and median. The measurement configuration consists of a spectrum analyzer, a biconical antenna and a laptop computer. The program in the computer controls the spectrum analyzer through HP-IB port. The spectrum analyzer parameters such as resolution bandwidth, video bandwidth, frequency span and sweep time are assigned through the Laptop PC. Three kinds of stations were recorded depending on the transmitted information. One that transmits only music, one that transmits only speech (voice) and one that transmits the combination of these two.

The statistical evaluation of the previous measurements and the derivation of a closed-form mathematical expression have been studied in [5]. Further statistical processing showed that the Gaussian fit for the estimation of the PSD of the FM channel reciprocates very well to all the conditions of the transmitted data. The equation is given by

\[
\hat{G}_m(f) = y_o + A \exp \left\{ \frac{(f-f_m)^2}{2\omega^2} \right\}
\]

where the parameters \(y_o, A, f_m\) and \(\omega\) are given in the following Table 1. Fig. 1 depicts the mean recorded values and the Gaussian curve fit.

The total FM interference from all FM stations will then be the sum of their PSD’s

\[
S_i(f) = \sum_{m=1}^{M} \hat{G}_m(f) = M y_o + A \sum_{m=1}^{M} \exp \left\{ \frac{(f-f_i-(m-1)\Delta f_m)^2}{2\omega^2} \right\}
\]

where \(M\) is the number of the FM stations, \(y_o, A\) and \(\omega\) are given in Table 1, \(f_i\) is the frequency if the first FM station and \(\Delta f_m\) is the carrier separation between two subsequent stations.

<table>
<thead>
<tr>
<th>(y_o)</th>
<th>-94.20817 dBm</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A)</td>
<td>58.92433 dBm</td>
</tr>
<tr>
<td>(f_m)</td>
<td>100 MHz</td>
</tr>
<tr>
<td>(\omega)</td>
<td>0.09697 MHz</td>
</tr>
<tr>
<td>(R^2)</td>
<td>0.985</td>
</tr>
</tbody>
</table>

Table 1: Gaussian fit parameters

![Fig. 1 Mean value of a FM station and its spectrum approximation](image)
4 Interference Analysis

Consider the receiver model depicted in Fig. 2. The total received signal is corrupted by noise and interference

\[ r(t) = s(t) + i(t) + n(t) \]  

where \( i(t) \) denotes the interference and \( n(t) \) is the zero-mean white Gaussian noise. Assuming that code synchronization has been established, the input to the demodulator is

\[ r_1(t) = s(t)c(t) + i(t)c(t) + n(t)c(t) \]  

The input quantity to the decision device, during the period \( T_b \) of the data signal, is

\[ S = \int_0^{T_b} g_r(t)r_1(t)\cos(\omega_c t + \theta) dt \]  

(8)

We assumed that \( f_o = \omega_o / 2\pi \gg 1 / T_b \), so that the double frequency term is negligible, suppressed by the bandpass filter following the demodulator. From equations (6), (7), (8) and the fact that \( b(t) \) is constant over the period \( T_b \) we have

\[ S \approx \sqrt{2P} \int_0^{T_b} b(t)\sum_{n=0}^{G_r-1} p^2(t-nT_c)dt + S_i + S_n = \]

\[ = \pm \sqrt{2P}T_b \frac{g_r(t)}{2} + S_i + S_n \]  

where

\[ \int_0^{T_b} \sum_{n=0}^{G_r-1} p^2(t-nT_c)dt = G_p T_c = T_b \]  

(10)

\[ S_i = \int_0^{T_b} g_r(t)i(t)c(t)\cos(\omega_c t + \theta) dt \]  

(11)

\[ S_n = \int_0^{T_b} g_r(t)n(t)c(t)\cos(\omega_c t + \theta) dt \]  

(12)

The signal \( c(t) \) is the output of the PN code generator, that over a bit interval, is expressed as

\[ c(t) = \sum_{n=0}^{G_c-1} c_n p(t-nT_c)dt \]  

(13)

The code chip sequence \( \{c_n\} \) can be modeled as a random binary sequence, comprises by statistically independent symbols with equal probability, is uncorrelated (white) and therefore \( E[c_n c_m] = E[c_n]E[c_m] \) for \( n \neq m \) and these conditions imply that \( E[c_n] = 0 \) and \( E[c_n^2] = 1 \). Since they arise from different physical sources, it is assumed that \( i(t), n(t), c(t) \) and \( \theta \) are statistically independent of each other. Based on the previous results, it follows that \( E[S_i] = E[S_n] = 0 \). In DS spread spectrum applications, the binary sequence with elements \( \{0,1\} \) is mapped into a corresponding binary sequence with elements \( \{-1,1\} \). Suppose that a bit 1 is transmitted. The decision device produces the symbol 1 if \( S > 0 \) and the symbol 0 if \( S < 0 \). An error occurs if \( S < 0 \) when \( b(t) = +1 \) or if \( S > 0 \) when \( b(t) = -1 \). The probability that \( S = 0 \) is zero.

Substituting the (13) into (11), we obtain

\[ S_i = g_r(t)\sum_{n=0}^{G_r-1} c_n J_n \]  

(14)

where

\[ J_n = \int_{nT_c}^{(n+1)T_c} i(t)p(t-nT_c)\cos(\omega_c t + \theta) dt \]  

(15)

Similarly

\[ S_n = g_r(t)\sum_{n=0}^{G_r-1} c_n N_n \]  

(16)

\[ N_n = \int_{nT_c}^{(n+1)T_c} n(t)p(t-nT_c)\cos(\omega_c t + \theta) dt \]  

(17)

The equation (9) can also be written as \( S \cong S_o + y_i \) where \( S_o \) is the desired signal and \( y_i = S_i + S_n \) denotes the total additive interference plus Gaussian noise. The first term of the right-hand side of the equation is deterministic and its value is given by the equation (9). Because the terms of \( S_i \) are independent random variables with zero mean value uniformly bounded and \( \text{var}[S_i] \to \infty \) with \( G_p \to \infty \), applying the Central Limit Theorem (CLT) \( [6] \) implies that \( S_i \) converges in a Gaussian distribution.
with zero mean value and variance 1. Consequently, the distribution of $S_i$ is almost Gaussian when $G_p$ is large. We must underline, that for the present analysis long sequences have been utilized with large processing gain. Therefore, the second term is a summation of two zero-mean independent Gaussian variables, indicating that $y_i$ will have an almost zero-mean Gaussian distribution and variance given by $\text{var}\{y_i\} = \text{var}\{S_i\} + \text{var}\{S_n\}$.

Based on CLT and Gaussian approximation, the probability of error will be given by [7]

$$P_e = \frac{1}{2} \text{erfc} \left[ \frac{E_{S|T_0}}{\sqrt{\text{var}\{S_n\} + \text{var}\{S_i\}}} \right]$$

and the total signal-to-noise ratio at the output of the correlator by

$$\text{SNR}_0 = \frac{E^2[S]}{\text{var}\{S_n\} + \text{var}\{S_i\}}$$

where $E_b = P T_0$ is the received bit energy $b(t)$, $P$ is its mean power and $\text{erfc}$ is the complementary error function defined by $\text{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^\infty \exp(-y^2)dy$

Based on the previous definitions, the variance of the variable $y_i$ is

$$\sigma^2_{y_i} = \text{var}\{y_i\} = \text{var}\{S_i\} + \text{var}\{S_n\} = E\left[S_i^2\right] + E\left[S_n^2\right]$$

$$= g_1^2(t) \sum_{n=0}^{G_i-1} \sum_{m=0}^{G_p-1} E[c_n c_m] E\left[J_n J_m\right] + g_1^2(t) \sum_{n=0}^{G_i-1} \sum_{m=0}^{G_p-1} E[c_n c_m] E\left[N_n N_m\right]$$

$$= g_1^2(t) \sum_{n=0}^{G_i-1} E\left[J_n^2\right] + g_1^2(t) \sum_{n=0}^{G_p-1} E\left[N_n^2\right]$$

$$= g_1^2(t) G_p E\left[J_n^2\right] + g_1^2(t) G_p E\left[N_n^2\right]$$

(20)

where we have used $E[c_n c_m] = \delta_{nm}$. Suppose that $\theta$ is an independent random variable uniformly distributed in over $[0, 2\pi]$. The stationary of $n(t)$ and a change of variables implies that the variance of $N_a$ is

$$E\left[N_a^2\right] =$$

$$= E\left[ \int_{-\infty}^{\infty} n(t)n(\lambda) p(t) p(\lambda) \cos(\omega_o t + \theta) \cos(\omega_o \lambda + \theta) dtd\lambda \right]$$

$$= \int_{-\infty}^{\infty} R(t-\lambda) p(t) p(\lambda) \cos(\omega_o t + \theta) \cos(\omega_o \lambda + \theta) dtdt =$$

$$= \frac{N_o}{4} \int_{-\infty}^{\infty} [1 + \cos(2\omega_o t + 2\theta)] dt = \frac{N_o T_c}{4}$$

(21)

where we have used the autocorrelation function for the AWGN $R_s(t-\lambda) = E\left[n(t)n(\lambda)\right] = \frac{N_o}{2} \delta(t-\lambda)$.

$N_o/2$ is the two-sided noise power spectral density and we assumed that $f_o = \omega_o / 2\pi \gg 1/T_c$.

For the calculation of the variance of interference, we will follow the noise procedure and can result to

$$E\left[J_s^2\right] = \frac{1}{2} \int_{-T_c}^{T_c} R(t-\lambda) p(t) p(\lambda) \cos(\omega_o (t-\lambda)) dtd\lambda$$

(22)

where $R(t)$ is the autocorrelation of $i(t)$ and we have neglected the double frequency term. A change of variables, using $\tau = t-\lambda \text{ and } s = t + \lambda$, the double integral can be reduced to single, then

$$E\left[J_s^2\right] = \frac{1}{2} \int_{-T_c}^{T_c} R(\tau) R_{\text{p}}(\tau) \cos(\omega_o \tau) d\tau$$

(23)

where $R_\text{p}(\tau)$ is the autocorrelation function of the PN sequence given by equation (2) and its PSD $S_{\text{p}}(f)$ given by equation (3). The limits of the integral can be extended to infinity because the integrand is truncated. Because $R(t)$ is an even function, the convolution theorem and the known Fourier transform of $R_\text{p}(\tau)$, yield the following [8]

$$E\left[J_s^2\right] = \frac{1}{2} T_c \int_{-\infty}^{\infty} S_\text{c}(f) \sin^2\left[\left(f-f_o\right)T_c\right] df$$

(24)

where $S_\text{c}(f)$ is the PSD of the interference after passage through the wideband bandpass filter. If $S_\text{c}(f)$ is the PSD of the interference at the input of the wideband bandpass filter and $H(f)$ is its transfer function, then $S_\text{c}(f) = S_\text{c}(f) |H(f)|^2$.

Suppose that the effects of the wideband filter and the integration over negative frequencies are negligible ($f_o \gg 1/T_c$), then

$$E\left[J_s^2\right] = \frac{1}{2} T_c \int_{f_o-W/2}^{f_o+W/2} S_\text{c}(f) \sin^2\left[\left(f-f_o\right)T_c\right] df$$

(25)

where we have assumed that the bandwidth of the
\[ P_e = \frac{1}{2} \text{erfc} \left( \frac{E_b}{N_o} \frac{g_r^2(t)}{4} \frac{T_b}{2} \frac{f_b + f_r T_r}{G_p f_b W_i/2} \int S_i(f) \sin \left[ \left( f - f_r \right) T_r \right] df \right) ^{1/2} \]

interference is \( W_i \leq W = \frac{2}{T_c} \). The PSD of the interference \( S_i(f) \) is given by equation (5) in the previous section and the total probability of error is given by equation (26).

5 Numerical Results

For the calculation of the probability of error we applied the CLT for the estimation of the total power spectrum of all FM stations, implementing the mean value by \( f_w = \frac{f_1 + f_2 + \ldots + f_M}{M} \) and standard deviation \( \omega_w = M \omega \). We assumed that all the FM stations \( M \) have the same standard deviation and thus power. For best results, we must consider as many stations as they can fit in the FM frequency band.

Suggestively, we show the results in Fig. 3 to Fig. 5 applying equation (26) for \( g_r(t) = 2 \), \( G_p = 50 = 17 \text{dB} \) and \( \Delta f_o = 500 \text{kHz} \) for the probability of error versus \( E_b / N_o \) for various signal-to-interference ratios \( P/P_i \), chip rate \( f_c \), FM stations \( M \) and frequency differences \( \Delta f \) between the two systems.

5 Conclusion

Following the CLT and the Gaussian approximation for the derivation of the total FM interference, we can conclude to the following results:
- The worst case for the probability of error occurs when the two systems have identically frequencies.
- The frequency difference \( \Delta f \) has minimal impact to the performance of the system, while the number
of FM stations is sufficiently large.
- As the total number of the FM stations is kept large considering small frequency separation $\Delta f_m$, the curves of bit-error-rate are improved.

References: